A NOTE ON THE CONSTRUCTION OF MODIFIED CUBIC DESIGNS

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1. Introduction

In this paper a method of construction of the Modified Cubic (M.C.) Designs defined by Singla [3] is given. This method is an extension of the method of differences used by Raghavarao [2] for the construction of $L_2(s)$ designs. For the definitions of statistical terms used here we refer to Raghavarao [1].

2. M. C. Designs

Singal [3] made the following definition.

2.1. Definition. A *PBIB* design with three associate classes is said to be a M.C. design if it statisfies the following association scheme.

Let there be $v=s^3$ treatments, s an integer, treatments being denoted by (α, β, γ), $(\alpha, \beta, \gamma)=1, 2, \ldots, s$.

Two treatments (α, β, γ) and $(\alpha', \beta', \gamma')$ are

(*a*) first associates if

$$\alpha \neq \alpha', \ \beta=\beta', \ \gamma=\gamma' ;$$

(*b*) second associates if

$$\alpha \neq \alpha', \ \beta=\beta', \ \gamma \neq \gamma'$$

or (*ii*) $\alpha=\alpha', \ \beta \neq \beta' \ \gamma=\gamma'$

or (*iii*) $\alpha=\alpha', \ \beta=\beta', \ \gamma \neq \gamma'$

or (*iv*) $\alpha \neq \alpha', \ \beta \neq \beta', \ \gamma=\gamma'$ ...(2·1)

(*c*) and third associates otherwise.

Obviously for this ossociation scheme

$$n_1=s-1, \ n_2=2s(s-1), \ n_3=s(s-1)^2.$$

3. Construction of M.C. Designs

Let M be a module of s elements, $0, 1, 2, \ldots, s-1$. To each element U of M we associate s^2 symbols denoted by U_{xy}.

$$x, y=0, 1, 2 \ldots s-1 ;$$

the treatment U_{xy} is said to belong to the class xy. There will thus be s^2 classes in all.

We define,

$$U_{xy} - U'_{x'y'} = (U - U')_{xy\,x'y'}$$

where $(U-U')$ is an element of Mod(s).

Now we have the following theorem:

Theorem 3.1. If we can find a set of t initial blocks of size k such that—

(*i*) treatments belonging to different classes occur the same number say r times in the initial sets ;

(*ii*) treatments in any set are distinct ;

(*iii*) the differences for which $x=x'$, $y=y'$ occur λ_1 times, the differences for which $x=x'$, $y\neq y'$ or $x\neq x'$, $y=y'$ occur λ_2 times and the other differences occur λ_3 times.

Then by developing these t initial blocks we shall get a M.C. design with parameters

$$v=s^3,\ b=st,\ r,\ k,\ \lambda_1,\ \lambda_2 \text{ and } \lambda_3 \qquad \text{...(3.1)}$$

Proof. Since there are s^2 classes in all and each class occurs r times in the initial sets therefore,

$rs^2=kt$ (Total number of plots in the initial sets). ...(3.2)

From (3.1) $\quad t=b/s$, giving

$$rs^2=kb/s$$

so that $\quad b=st$.

If we denote the treatment $\alpha\beta\gamma$ by $\alpha\beta\gamma$ then it is clear that the number of treatments for which

$$\beta=\beta',\ \gamma=\gamma' \text{ is } n_1,$$

the number of treatments for which

$$\beta\neq\beta',\ \gamma=\gamma'$$

or $\quad \beta=\beta',\ \gamma\neq\gamma'$ is n_2

and the number of treatments for which $\beta\neq\beta'$, $\gamma\neq\gamma'$ is n_3.

Hence the theorem.

3.1. Example. Let there be $v=s^3=27$ treatments denoted by the triplets (i, j, k), $i, j, k=0, 1, 2$.

For the purpose of construction suppose the treatment (α, β, γ) is written as $(\alpha_{\beta\gamma})$ (β, γ being suffixes of α).

The following 9 blocks satisfy the conditions of the above theorem.

0_{10}	0_{20}	0_{01}	0_{02}	1_{01}	1_{02}	1_{10}	1_{20}	2_{10}	2_{20}	2_{01}	2_{02}
0_{00}	0_{02}	0_{11}	0_{21}	1_{00}	1_{02}	1_{11}	1_{21}	2_{00}	2_{02}	2_{11}	2_{21}
0_{00}	0_{01}	0_{12}	0_{22}	1_{00}	1_{01}	1_{12}	1_{22}	2_{00}	2_{01}	2_{12}	2_{22}
0_{11}	0_{12}	0_{00}	0_{20}	1_{00}	1_{20}	1_{11}	1_{12}	2_{00}	2_{20}	2_{11}	2_{12}
0_{01}	0_{21}	0_{10}	0_{12}	1_{01}	1_{21}	1_{10}	1_{12}	2_{01}	2_{21}	2_{10}	2_{12}
0_{02}	0_{22}	0_{10}	0_{11}	1_{02}	1_{22}	1_{01}	1_{11}	2_{02}	2_{22}	2_{10}	2_{11}
0_{00}	0_{10}	0_{21}	0_{22}	1_{00}	1_{10}	1_{21}	1_{22}	2_{00}	2_{10}	2_{21}	2_{22}
0_{20}	0_{22}	0_{01}	0_{11}	1_{01}	1_{11}	1_{20}	1_{22}	2_{01}	2_{11}	2_{20}	2_{22}
0_{20}	0_{21}	0_{02}	0_{12}	1_{02}	1_{12}	1_{20}	1_{21}	2_{02}	2_{12}	2_{20}	2_{21}

These 9 blocks when developed give a M.C. design with parameters

$$v=27=b,\ r=k=12,$$
$$\lambda_1=12,\ \lambda_2=3,\ \lambda_3=6.$$

It may be noted that these 9 blocks when developed repeat themselves. Therefore, these 9 blocks themselves form a M.C. design with parameters

$$v=27,\ b=9,\ k=12,\ r=4,$$
$$\lambda_1=4,\ \lambda_2=1,\ \lambda_3=2.$$

SUMMARY

In this paper the method of differences given by Raghavarao [2] for the construction of $L_2\,(s)$ designs has been extended for the construction of M.C. designs.

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A NOTE ON UNEQUAL PROBABILITY SAMPLING

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1. INTRODUCTION

Among the probability proportional to size without replacement (*wor*) sampling schemes proposed so far, Yates and Grundy's (1953) scheme, for $n=2$, is as follows : Select first unit out of N units with probabilities proportional to size

$$p_i \left(=\frac{X_i}{X}\right),$$

X_i being the size of the ith unit and

$$X=\sum_{i=1}^{N} X_i,$$

and the second unit out of the remaining $(N-1)$ units with probabilities proportional to sizes of the remaining units,

$$p'_j \left(=\frac{X_j}{X'}\right),$$

X' being the total of the sizes of the remaining $(N-1)$ units. For this scheme the probability of selecting unit i and that of the pair i and j of units are given by

$$\Pi_i=p_i\left(1+A-\frac{p_i}{1-p_i}\right),$$

where
$$A=\sum_{i=1}^{N} \frac{p_i}{1-p_i} \qquad \ldots(1)$$

and
$$\Pi_{ij}=p_ip_j\left(\frac{1}{1-p_i}+\frac{1}{1-p_j}\right)$$

respectively.

As pointed out by them this scheme of sampling is not equivalent to selecting a pair of units with replacement with probabilities proportional to sizes and rejecting those samples in which the same unit is repeated. A sampling scheme has been proposed here which gives the same variance of $\hat{Y}_{HT}$ (Horvitz Thompson estimator of population total) as Yates and Grundy's Scheme.

2. Scheme Proposed

Select two units with replacement, one with probabilities

$$p_j \left(= \frac{X_j}{X} \right),$$

X_j being the size of the jth unit and

$$X = \sum_{j=1}^{N} X_j,$$

and the other with revised probabilities

$$p_j^* = \frac{p_j(1-p_j)^{-1}}{\left[\sum_{j=1}^{N} p_j(1-p_j)^{-1} \right]}.$$

Repeat the procedure until two different units are selected.

It can be easily seen that for the proposed scheme the probability of inclusion of the ith unit and that of the joint inclusion of the pair i and j of units in a sample of size two are given by

$$\Pi_i = \left(p_i^* \sum_{j \neq i}^{N} p_j + p_i \sum_{j \neq i}^{N} p_j^* \right) \times \left(1 - \sum_{i=1}^{N} p_i p_i^* \right)^{-1}$$

$$= p_i + p_i \sum_{j \neq i}^{N} \frac{p_j}{1-p_j}$$

$$= p_i + p_i \left(A - \frac{p_i}{1-p_i} \right)$$

$$= p_i \left[1 + A - \frac{p_i}{1-p_i} \right]$$

and
$$\Pi_{ij} = p_i p_j \left[\frac{1}{1-p_i} + \frac{1}{1-p_j} \right].$$

respectively.

The above expressions for Π_i and Π_{ij} are the same as those given in (1) for the Yates and Grundy's sampling scheme. Since the variance of $\hat{Y}_{HT}$ involves only Π_i's and Π_{ij}'s apart from the unknown Y_i's, the values of the character under study, the proposed sampling scheme will also give the same variance of $\hat{Y}_{HT}$ as is obtained under Yates and Grundy's sampling scheme.

SUMMARY

In this note a scheme of unequal probability sampling without replacement (*wor*) for samples of size two has been suggested. It is noted that for this sampling scheme the inclusion probabilities for a unit and a pair of units are the same respectively as the corresponding probabilities for Yates and Grundy's (1953) scheme.

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REFERENCE

[1] Yates, F. and Grundy, P.M. (1953) : "Selection *wor* from within strata with probabilities proportional to size Jour. Royal Statist. Soc. B 15, 253-61.